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PHYSICS-INFORMED NEURAL NETWORKS FOR SOLVING PHYSICAL PARTIAL DIFFERENTIAL EQUATIONS IN AEROSPACE ENGINEERING

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Abstract. *This analytical review evaluates Physics-Informed Neural Networks (PINNs) for solving PDEs in aerospace engineering through a literature-based comparison on standard benchmarks (cylinder, NACA0012, inverse Burgers). It summarizes methodological advances—Volume-Weighted PINNs, gradient-enhanced losses, hard constraints, transfer learning, geometry-aware and isogeometric schemes—and notes reported accuracy gains (VW-PINNs: surface pressure errors ≈1–3%; viscosity ≈1.5%). Limitation: no original numerical experiments; further validation in multiscale/high-Re regimes is required.*
Keywords: *PINNs; methodological advances; volume-weighted losses; gradient-enhanced PINNs; computational aerodynamics; inverse problems; benchmark cases*

Introduction. Physics-Informed Neural Networks (PINNs) have emerged as a distinct class of methods at the intersection of computational fluid/aerodynamics and deep learning. The canonical formulation of PINNs for forward and inverse problems was introduced in (Raissi et al., 2019) [7]. The central idea is to embed governing physical equations and problem constraints directly into the loss function, enabling PDE solution approximation without an explicit space–time mesh and with the integration of experimental or field data (Luo et al., 2025) [4]. The balance of loss components and the fusion of data with physical constraints have been demonstrated in applied studies (Guo et al., 2020) [2]. *Aim and Objectives*. The study aims to synthesize recent advances in Physics-Informed Neural Network (PINN) research and identify pragmatic improvements and open questions relevant to computational aerodynamics. To achieve this goal, the research (1) systematizes contemporary PINN variants, including gradient-enhanced and volume-weighted formulations (gPINNs, VW-PINNs), and classifies their algorithmic features; (2) analyzes methods for balancing loss components and enforcing physical constraints, such as weighting strategies, gradient-based corrections, hard constraints, and weak formulations; and (3) compares reported accuracy and convergence trends with baseline CFD results on canonical benchmarks (cylinder flow, NACA0012 airfoil, inverse Burgers problem) to reveal methodological limitations requiring further validation.

The main part. *Methodology*. This study considers the application of PINNs in aerospace engineering with the aim of analyzing the effectiveness of these methods in solving partial differential equations (PDEs). To prepare the material, publications from the Web of Science, Scopus, and Google Scholar databases were reviewed for the period 2019–2025, covering both fundamental works (Raissi et al., 2019) [7] and the latest methodological advancements (Song et al., 2024; Ren et al., 2025) [10, 8]. The main directions of analysis are related to the formulation of PINNs. Both forward and inverse problems are examined, where PDEs, boundary, and initial conditions are integrated into the loss function. Methods for balancing loss components are studied, including the use of weighting coefficients and enhanced variants of PINNs, such as gPINNs (gradient-enhanced PINNs) and VW-PINNs (Volume-Weighted PINNs). In addition, the use of hard constraints, transfer learning, weak-form formulations, and isogeometric parameterization for complex geometries is reviewed, along with the validation of methods conducted on benchmark problems such as cylinder flow in viscous

and inviscid regimes, NACA0012 airfoil profiles under different angles of attack and flow conditions, and the inverse Burgers' equation problem. The inclusion criteria considered were articles demonstrating the implementation of PINNs in aerodynamics, fluid mechanics, and related disciplines. The exclusion criteria were publications without original numerical or experimental results, as well as studies lacking physically relevant data. Each work was analyzed in terms of relevance and novelty, particularly with respect to the current state of development of PINNs (the presence of enhancements and methodological innovations), as well as the practical significance for aerospace engineering (the ability of the approach to ensure accuracy and convergence in aerodynamic and structural analysis problems). It is also necessary to outline the limitations: despite rapid progress, PINNs—and VW-PINNs in particular—remain a relatively new approach, and therefore the number of verified sources is still limited. Nevertheless, the results of numerical experiments (Song et al., 2024; Ren et al., 2025; Mustajab et al., 2024) [10, 8, 5] confirm the promise of VW-PINNs for applications in computational aerodynamics and the design of aerospace vehicles.

Results and Discussion. The solution $u(x,t)$is approximated by a neural network with parameters θ; automatic differentiation evaluates the residuals of differential operators. This setup enables a unified treatment of forward and inverse nonlinear PDE problems (Raissi et al., 2019) [7]. The standard loss includes contributions from: PDE residuals at interior collocation points; boundary conditions; initial conditions; observational data.

Validation on *cylinder flow* in an *inviscid, compressible regime* showed a reduction of the loss function by seven orders of magnitude, while standard PINNs achieved only three (Song et al., 2024) [10]. At the same time, the relative wall-pressure error was only 1.33 % compared to the results obtained with the finite-volume method. In the case of *cylinder flow* at *Reynolds number Re = 40*, VW-PINNs achieved a pressure coefficient error of 0.53 % and correctly reproduced the *vortex shedding* process, whereas classical PINNs exhibited stagnation with significant residual errors (Raissi et al., 2019) [7].

For the *NACA0012 airfoil* at *a 5° angle of attack*, VW-PINNs produced consistent velocity and pressure fields with a pressure coefficient error of 3.21 %, while standard networks led to systematic deviations (Ren et al., 2025) [8]. In the *inverse Burgers' equation problem*, the viscosity estimation error decreased to 1.48 %, whereas standard PINNs yielded a much worse result — about 19.82 %. The numerical results obtained confirm the effectiveness of the VW-PINNs approach for solving aerodynamic problems, especially in the design of aerospace vehicles. Overall, the findings indicate that relative errors in determining the pressure coefficient on object surfaces are within 1–3 % compared to baseline CFD methods, which confirms the competitiveness of the approach (Mustajab et al., 2024) [5].

Therefore, two particularly effective directions can be identified for the aerospace field: (I) gPINNs (gradient-enhanced losses) — losses with derivatives that improve stability in parabolic and convection–diffusion equations by adding derivative residuals (Sun & Feng, 2023) [11]; (II) VW-PINNs — normalization of residuals by local volumes inversely proportional to sample density, which balances convergence rates across the entire domain (Song et al., 2024) [10].

The key advantages of the VW-PINNs approach for aerodynamic applications can be outlined as follows. *First*, the mesh-free formulation and the ability to integrate data naturally combine experimental observations with physical models of the Navier–Stokes, Euler, or even complex aeroelastic equations (Chen et al., 2023; Luo et al., 2025) [1, 4]. *Second*, VW-PINNs demonstrate robustness to issues related to non-uniform discretization, ensuring balanced convergence even in regions with sharp gradients, shock waves, or flow separation (Song et al., 2024) [10]. *Third*, the method shows

compatibility with adaptive sampling strategies, enabling a reduction in training time without loss of accuracy (Guo et al., 2020) [2]. In contrast to traditional CFD approaches based on spectral methods, PINNs reduce the cost of mesh generation and naturally combine forward and inverse formulations (Sun & Feng, 2023) [11].

Recent reviews summarize these advantages and limitations in the context of applying these methods to realistic problems (Ren et al., 2025) [8], while emphasizing challenges of scalability, multiscale dynamics, and training stability for stiff PDEs (Luo et al., 2025) [4]. Promising directions include multi-frequency bases, weak-form formulations, and hybridization with continuum mechanics methods (Ren et al., 2025) [8]. Hard parameterizations reduce boundary violations and improve convergence (Chen et al., 2023) [1], whereas transfer learning increases training efficiency for high-frequency and multiscale regimes (Mustajab et al., 2024) [5]. Weighted coefficients balance the components of the loss function and affect both convergence and accuracy (Song et al., 2024) [10].

Uneven sampling biases the optimizer: residual averaging overweights dense regions and neglects sparse ones, causing ill-conditioned losses and under-convergence of global PDE residuals (Song et al., 2024) [10]. This is especially critical in aerodynamic flows with shocks, separations, or steep boundary gradients. Even with weight normalization, typical problems remain at high Reynolds numbers (Sun & Feng, 2023) [11], in strongly compressible flows, and in multiscale regimes, where standard MLPs lack sufficient expressive capacity. Reviews highlight the need for larger batch sizes, improved training strategies, and architectures with built-in multiscale representations or weak-form formulations (Luo et al., 2025) [4].

For complex curved geometries, promising approaches include geometry-aware surrogate models for the Navier–Stokes equations (Oldenburg et al., 2022) [6], high-order isogeometric methods for CAD-conforming domain parameterization (Schneider et al., 2021) [9], and weak-form schemes for geometrically exact 3D beams (Weeger et al., 2022) [12]. Theoretical error estimates and generalization results for nonlinear PDEs in realistic geometries are still under development, although initial progress has been made for certain classes of equations and domain decompositions (Sun & Feng, 2023) [11].

Literature reviews point to the necessity of employing extended architectures, including models with multi-frequency features, variational blocks, weak-form methods, and geometry-aware surrogates (Ren et al., 2025; Oldenburg et al., 2022) [8, 6]. For complex geometries, promising directions include the use of isogeometric methods that ensure correct CAD parameterization, as well as weak-form schemes for three-dimensional structures (Schneider et al., 2021; Weeger et al., 2022) [9, 12]. At the same time, the development of rigorous error estimates and generalization guarantees for realistic geometries and complex PDE classes remains an active research area (Li et al., 2024) [3].

Conclusions. The conducted analytical review achieved its primary aim—to synthesize recent advances in Physics-Informed Neural Networks (PINNs) and to identify practical improvements and open questions relevant to computational aerodynamics. *First*, the systematization of contemporary PINN variants revealed that gradient-enhanced (gPINNs) and volume-weighted (VW-PINNs) formulations constitute the most consistent strategies for stabilizing loss function minimization and improving convergence on aerodynamic benchmarks. *Second*, the comparative analysis of loss-balancing and constraint-enforcing methods demonstrated that adaptive weighting, gradient corrections, and hard parameterizations substantially reduce residual bias and boundary errors, particularly in flow configurations with shocks or separations. *Third*, a cross-reference of published numerical results confirmed that VW-PINNs achieve 1–3% accuracy relative to reference CFD data for cylinder and NACA0012 airfoil flows, while in the inverse Burgers' problem the viscosity error was reduced from approximately 20% to below 2%. These findings validate the efficiency of the VW-PINN framework

in canonical aerodynamic tests. Despite this progress, challenges persist in high-compressibility and multiscale regimes, where conventional multilayer perceptrons show limited representational capacity. Addressing these limitations requires enhanced architectures that incorporate multi-frequency and weak-form elements, geometry-aware surrogates, and isogeometric formulations. Equally important is the development of verifiable error estimates and generalization metrics for realistic geometries and stiff nonlinear PDEs.

In perspective, VW-PINNs can be viewed not merely as an incremental improvement but as a transformative framework for computational aerodynamics. Their ability to unify physical modeling, data assimilation, and inverse analysis provides a pathway toward physics-grounded, data-efficient modeling strategies for future aerodynamic research and design optimization.

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